Functional Approaches to Programming

~ Evaluation and Scoping ~

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Outline

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Plan

Evaluation Techniques

- Strict Evaluation / Applicative Order
- Lazy Evaluation / Normal Order
- Comparative Merits

Scoping Forms

- Block Structure / Locality
- Lexical *vs.* Dynamic Scoping

Plan

Strict Evaluation (Lisp)

- *A.k.a.* "applicative order"
- **Process**
  1. Evaluate the sub-expressions from left to right
  2. Apply the operator to its arguments

  Recursive process ("tree accumulation")
- **Leaves:** litterals / primitive operators
- **Environment**
  - Abstract expressions values
  - Special case: primitive operators

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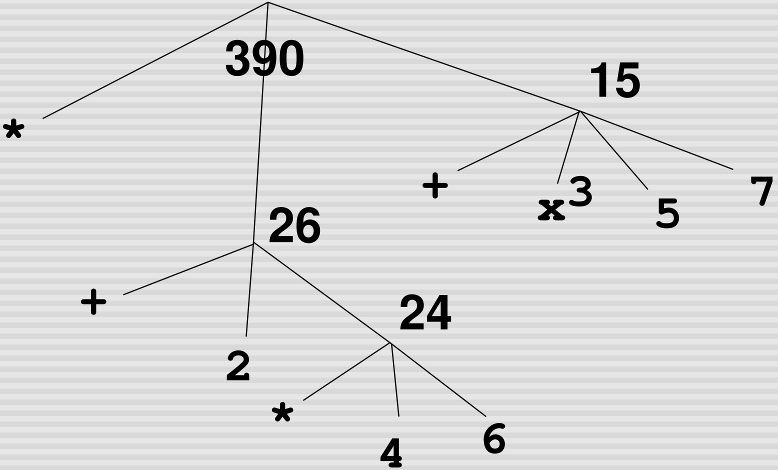
Exemple

```
(* (+ 2 (* 4 6)) (+ x 5 7))
```



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Substitution Model

- **For abstract expressions**
  - Identical to the previous case
  - *Substitution* (preliminary step)
    Replacement of formal parameters with their corresponding value
- **Remarks**
  - Theoretical model
  - Substitution $\Rightarrow$ local environment
  - ⚠ Complicated! *Cf.* λ-calcul, α-conversion, name collision

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Example

```
(defun sq (x) (* x x))

(defun ssq (x y) (+ (sq x) (sq y)))

(defun f (a)
  (ssq (+ a 1) (* a 2)))
```

```
(f 5)
(ssq (+ a 1) (* a 2))
(ssq (+ 5 1) (* 5 2))
(ssq 6 (* 5 2))
(ssq 6 10)
(+ (sq x) (sq y))
(+ (sq 6) (sq 10))
(+ (* x x) (sq 10))
(+ (* 6 6) (sq 10))
(+ 36 (sq 10))
(+ 36 (* x x))
(+ 36 (* 10 10))
(+ 36 100)
136
```

Special Operators

- **Problem:** non strict idioms
  Conditionals, boolean logic, etc.
- **Solution:** special *primitive* operators
  Particular evaluation technique
- 25 in Lisp (`setq`, `if`, `quote`, `function`, *etc.*)
- Test: `(special-operator-p 'if)`

Plan

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Lazy Evaluation (Haskell)

- *A.k.a.* "normal order"
- **Process**
  - Same substitution model
  - But *on-demand* evaluation
- **Performance:** inefficient model
  Work on an evaluation graph ("memoization")

⚠ **Constraint:** purely functional only

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Example

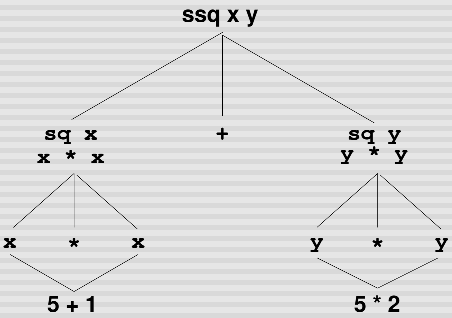
```haskell
sq :: Float -> Float
sq x = x * x

ssq :: Float -> Float -> Float
ssq x y = sq x + sq y

f :: Float -> Float
f a = ssq (a + 1) (a * 2)
```

```
f 5
ssq (a + 1) (a * 2)
ssq (5 + 1) (5 * 2)
sq x + sq y
sq (5 + 1) + sq (5 * 2)
(x * x) + (y * y)
(5 + 1) * (5 + 1) + (5 * 2) * (5 * 2)
6 * (5 + 1) + (5 * 2) * (5 * 2)
6 * 6 + (5 * 2) * (5 * 2)
36 + (5 * 2) * (5 * 2)
36 + 10 * (5 * 2)
36 + 10 * 10
36 + 100
136
```

Evaluation Graph / Memoization




```
ssq (5 + 1) (5 * 2)
sq (5 + 1) + sq (5 * 2)
(5 + 1) * (5 + 1) + (5 * 2) * (5 * 2)
6 * 6 + 10 * 10
```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Equation Evaluation

```
f p1 p2 p3 ... = e1
f q1 q2 q3 ... = e2
...
```

- **Selected equation:** first “match”
- **Matching**
  - Only the arguments necessary to the decision process
  - Only the arguments *parts* necessary to the decision process

Examples

Minimal evaluation of the arguments

```haskell
foo :: Float
foo = 2 * foo

prod :: Float -> Float -> Float
prod x 0 = 0
prod 0 y = 0
prod x y = x * y
```

```
prod 3 4 => (3) 12.0
prod 4 0 => (1)  0.0
prod foo 0 => (1) 0.0
prod 0 foo => Stack overflow
```

Partial evaluation of the arguments

```haskell
sh :: [Int] -> [Int] -> Int
sh [] ys = 0
sh (x:xs) [] = 0
sh (x:xs) (y:ys) = x + y
```

```
sh [1..3] [5..8]
(1) => sh (1:[2..3]) [5..8]
(2) => sh (1:[2..3]) (5:[6..8])
(3) => 1 + 5
    => 6
```


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Guards Evaluation

```haskell
max3 :: Int -> Int -> Int -> Int
max3 m n p
    | (m >= n) && (m >= p) = m
    | (n >= m) && (n >= p) = n
    | otherwise = p
```

```
max3 (2+3) (4-1) (3+9)
(1) => (2+3)>=(4-1) && (2+3)>=(3+9)
    =>      5>=(4-1) &&      5>=(3+9)
    =>      5>=3     &&      5>=(3+9)
    =>      True     &&      5>=(3+9)
    =>      True     &&      5>=12
    =>      True     &&      False
    => False
(2) => 3>=5  && 3>=12
    => False && 3>=12
    => False
(3) => 12
```

nrsqrt IV, Apocalypse

- Reminder

Lisp

```
(defun intlist (s)                    ;; KO
  (cons s (intlist (1+ s))))
```

Haskell

```
intlist :: Int -> [ Int ]             -- OK
intlist s = s : intlist (s + 1)
```

New view: work directly on the (infinite) Newton-Raphson sequence

Haskell

```
limit :: [ Float ] -> (Float -> Bool) -> Float
limit (y:ys) found
  | found y = y
  | otherwise = limit ys found

nrsqrt :: Float -> Float -> Float
nrsqrt x delta = limit (nrlist x 1) (nrfound x delta) -- partial application!

nrlist :: Float -> Float -> [ Float ]
nrlist x yn = yn : nrlist x ((yn + x / yn) / 2)

nrfound :: Float -> Float -> Float -> Bool
nrfound x delta yn = abs (x - yn * yn) <= delta
```

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Strict *vs.* Lazy Evaluation

- **Strict evaluation is more efficient**
  No computation redundancy, but *cf.* normal order + memoization
- **Lazy evaluation is more expressive**
  E.g. infinite data structures, but *cf.* Lisp macros

Haskell

```haskell
ifnot :: Bool -> a -> a -> a
ifnot test e1 e2 = if test then e2 else e1
```

Lisp

```lisp
(defmacro ifnot (test e1 e2)
  (list 'if test e2 e1))
```

- **No lazyness without purity, no impurity without determinism**
  But strict evaluation is not the only deterministic form
- **Church-Rosser** ("global confluence", *cf.* λ-calcul)
  Unicity of the canonical form, whatever the evaluation method (purely functional only)

Strict *vs.* Lazy Evaluation

- Semantic diff quizz

Lisp

```
(setq bar (sqrt (- (* 3 (+ 6 7)) 8)))
```

```
'(foo bar baz) ≠ (list foo bar baz)
```

```
(let* ((a (* x x)) ;; order important
       (b (+ a 1)))
  (+ (/ a b) (/ b a)))
```

Haskell

```
bar = sqrt (3 * (6 + 7) - 8)
```

```
[foo, bar, baz]
```

```
let b = a + 1 -- order not important
    a = x * x
in a / b + b / a
```


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Pseudo-Lazyness in Imperative

- Conditionals, *etc.*

C

```
if (something_that_turns_out_to_be_true)
{
  /* Computed */
}
else
{
  /* Not computed */
}
```


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Block Structure / Locality

- **Block:** set of $\{$*name* $\rightarrow$ *expression*$\}$ “bidings”
- **Evaluation environment:** nested blocks
- **Explicit *vs.* implici blocks**
  - `let`, `where`, `flet`, *etc.*
  - Global declarations, function parameters
- **Theoretical problems** (*cf.* α-conversion)

Lisp

```
(defun sq (x) (* x x))
(defun sq (y) (* y y))
```

```
(defun sq (x) (* x x))
(defun f  (x) (sq (* 2 x)))
```

Haskell

```
sq x = x * x
sq y = y * y
```

```
sq x = x * x
f  x =  sq (2 * x)
```


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Concept of Scoping

- **Context**
  - *Bound* variable: defined in the local block
  - *Free* variable: not locally defined
- **Scoping**
  - Binding lookup in the “nearest” block
  - But what do we mean by “near”?

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Scoping and Collision

- Local bindings take precedence

Lisp

```lisp
(defvar x 5)

(defvar y                ;; y = 8
  (+ (let ((x 3)) x)     ;; x = 3
     x))                 ;; x = 5
```

Haskell

```haskell
x :: Int
x = 5

y :: Int                -- y = 8
y = (let x = 3 in x)    -- x = 3
    + x                 -- x = 5
```

Scoping and Collision

⚠ But beware the language semantics!

- Lisp: local values computed *outside* of `let`

```lisp
(defvar x 2)

(defvar y
  (let ((x 3)         ;; x = 3
        (z (+ x 2))) ;; z = 2 + 2
    (* x z)))         ;; y = 3 * 4
```

- Nonsensical in Haskell (closer to Lisp's `let*`)

Lisp

```lisp
(defvar x 2)

(defvar y
  (let* ((x 3)         ;; x = 3
         (z (+ x 2))) ;; z = 3 + 2
    (* x z)))          ;; y = 3 * 5
```

Haskell

```haskell
x :: Int
x = 2

y :: Int
y = let x = 3      -- x = 3
        z = x + 2 -- z = 3 + 2
    in x * z      -- y = 3 * 5
```

Plan

Scoping Forms

- **Reminder:** "closest" binding lookup
  Deals with free variables only
- **Lexical (static):** lookup in the *definition* environment
- **Dynamic:** lookup in the *calling* environment
- ⚠ **Remark:** `let` does two different things at the same time!

Lisp

```
(let ((x 10)).               ;; lexical scope
  (defun foo () x))

(let ((x 20))
  (foo))                     ;; => 10
```

Lisp

```
(defvar x 10)                ;; dynamic scope
(defun foo () x)

(let ((x 20))
  (foo))                     ;; => 20
```


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Concept of Lexical Closure

- **Definition**
  Combination of a function and its defining environment
  Values of free variables at definition time
- **Advantages**
  - Foundation of the *whole* higher-order machinery!
  - *Cf.* λ-calcul / chapter 2

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Examples

- **Functional arguments**

Lisp

```lisp
(defun list+ (lst n)
  (mapcar (lambda (x) (+ x n)) lst))
```

Haskell

```haskell
(+++) :: [Int] -> Int -> [Int]
(+++) lst n = map (\x -> x + n) lst
```

- **Fonctional Returns**

```lisp
(defun make-adder (n)
  (lambda (x) (+ x n)))
```

```haskell
makeAdder :: Int -> Int -> Int
makeAdder n = \x -> x + n
```

- **Locality + mutation**

```lisp
(let ((cnt 0))
  (defun get-uid () (incf cnt))
  (defun reset-uids () (setq cnt 0)))
```

Dynamic Scoping (Lisp)

- Historical form in Lisp
- Since `Scheme`: lexical scoping
- Common Lisp: lexical scoping by default, dynamic possible
  - Global variables (`defvar`, *etc.*)
  - Local variables declared "special"

```lisp
(let ((x 10)).              ;; lexical scope
  (defun foo () x))

(let ((x 20))
  (foo))                    ;; => 10
```

```lisp
(defvar x 10)               ;; dynamic scope
(defun foo () x)

(let ((x 20))
  (foo))                    ;; => 20
```

```lisp
(let ((x 10))
  (defun foo ()
    (declare (special x)) ;; dynamic scope
    x))

(let ((x 20))
  (foo))                    ;; => 20
```


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Pro, or Anti Dynamic Scoping?

- **Advantages**
  - Several paradigms (Context-Oriented Programming, *etc.*)
  - Exception management (handlers)
  - Global variables (*e.g.* user options, *cf.* Emacs)

```lisp
(defvar *default-background* 'blue) ;; note the earmuffs!

(defun create-42-red-windows ()
  (let ((*default-background* 'red))
    (loop :repeat 42 :do (create-window))))
```

- **Drawbacks**
  - Unsuited to higher order
  - Origin of *very* nasty bugs
    Name collision problem
  - The very first example of higher order function from John McCarthy was wrong!

McCarthy's (bad) example

- First historical mapping function

```
(defmacro while (test &rest body)
  `(do () ((not ,test))
     ,@body))

(defun my-mapcar (func lst)
  (let (elt n)
    (while (setq elt (pop lst))
      (push (funcall func elt) n)) ;; <- name clash on n!!
    (nreverse n)))

(defun list+ (lst n)
  (my-mapcar (lambda (x)
               (declare (special n)) ;; Do that and...
               (+ x n)) ;; ... barf, name clash on n!!
             lst))
```


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